Hardness of Approximation meets Parameterized Complexity

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Day 1: The Setting

Day 2: Gap Creation

Day 3: Applications
Part 1: Hardness of Approximating MaxCover
- Recap
- MaxCover with Projection Property
- Gap Creation
**Part 1: Hardness of Approximating MaxCover**
- Recap
- MaxCover with Projection Property
- Gap Creation

**Part 2: Hardness of Approximating One-Sided Biclique**
- Recap
- Gap Creation
Part 1

Gap Creation in MaxCover
MaxCover: Recap

\[ \Gamma(U, W, E) \]

- Each \( W_i \) is a Right Super Node
- Each \( U_i \) is a Left Super Node

A labeling \( S \subseteq W \) of \( W \) covers \( U_i \) if

\[ \forall i \in [k], |S \cap W_i| = 1 \]

\( S \) covers \( U_i \) if

\[ \exists u \in U_i, \forall v \in S, (u, v) \in E \]

MaxCover(\( \Gamma \), \( S \)) = Fraction of \( U_i \)'s covered by \( S \)

MaxCover(\( \Gamma \)) = max \( S \) MaxCover(\( \Gamma \), \( S \))
MaxCover: Recap

Each $W_i$ is a **Right Super Node**
Each $U_i$ is a **Left Super Node**

$\Gamma(U, W, E)$

$\text{Determine if } \text{MaxCover}(\Gamma) = 1 \text{ or } \text{MaxCover}(\Gamma) \leq s$

Each $W_i$ is a Right Super Node
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MaxCover: Recap

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MaxCover($\Gamma$) = $\max_S$ MaxCover($\Gamma, S$)
MaxCover: Recap

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MaxCover($\Gamma$, $S$) = Fraction of $U_i$’s covered by $S$

MaxCover($\Gamma$) = $\max_S$ MaxCover($\Gamma$, $S$)

Determine if MaxCover($\Gamma$) = 1
or MaxCover($\Gamma$) $\leq s$
MaxCover: Projection Property

\[ \Gamma(U, W, E) \]

\( \Gamma \) has projection property:

For every \( U_i \) and \( W_j \),

- Induced subgraph of \((U_i, W_j)\) is:
  - complete bipartite graph (i.e., irrelevant), or,
  - \( \forall w \in W_j, \deg(w) = 1 \) (i.e., projection)
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MaxCover with Projection Property is \( \text{W}[1]\)-Hard

Input: \( G([n], E_0) \)

\[ \Gamma(U, W, E) \]

\[ U = \{ U_1, U_2, \ldots, U_k \} \]

\[ W = \{ W_1, W_2, \ldots, W_{\ell} \} \]

Parameterized Inapproximability

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MaxCover with Projection Property is $W[1]$-Hard

Input: \( G([n], E_0) \)

\[ U_i = [n] \text{ and } W_{j,j'} = E_0 \]
MaxCover with Projection Property is $W[1]$-Hard

Input: $G([n], E_0)$

$U_i = [n]$ and $W_{j,j'} = E_0$

$W_{j,j'}$ has projection to $U_j$ and $U_{j'}$
Inapproximability of MaxCover [K-LivniNavon’21]

There is a FPT reduction from MaxCover instance $\Gamma_0 = \left( U_0 = \bigcup_{j=1}^{r} U_j^0, W = \bigcup_{j=1}^{k} W_i, E_0 \right)$ with projection property to a MaxCover instance $\Gamma = \left( U = \bigcup_{j=1}^{O(\log |U_0|)} U_j, W = \bigcup_{j=1}^{k} W_i, E \right)$ such that

- If $\text{MaxCover}(\Gamma_0) = 1$ then $\text{MaxCover}(\Gamma) = 1$
- If $\text{MaxCover}(\Gamma_0) < 1$ then $\text{MaxCover}(\Gamma) \leq 0.75 |\Gamma|$

The reduction runs in time $2^{O(r \cdot |W| \cdot \log |U_0|)}$. 
Inapproximability of MaxCover [K-LivniNavon’21]

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- If $\text{MaxCover}(\Gamma_0) = 1$ then $\text{MaxCover}(\Gamma) = 1$
- If $\text{MaxCover}(\Gamma_0) < 1$ then $\text{MaxCover}(\Gamma) \leq 0.75$
- $|\Gamma| = \tilde{O}(2^r \cdot |W| \cdot \log |U_0|)$
- The reduction runs in time $2^{O(r)} \cdot \text{poly}(|\Gamma_0|)$. 
Coding Theory: Recap

- $C \subseteq [q]^L$
- **Distance** of $C$:

  $$\Delta(C) := \min_{x,y \in C} \| x - y \|_0$$
Coding Theory: Recap

- $C \subseteq [q]^L$

- **Distance** of $C$:

  $$\Delta(C) := \min_{x, y \in C} \|x - y\|_0$$

- For some constant $\rho > 0$, collection of $2^{\rho L}$ Random Binary Strings is a code with distance $L/4$
Coding Theory: Recap

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- **Distance** of $C$:
  \[
  \Delta(C) := \min_{x, y \in C} \|x - y\|_0
  \]

- For some constant $\rho > 0$, collection of $2^{\rho L}$ Random Binary Strings is a code with distance $L/4$

- **Reed Solomon Codes**:
  - Evaluations of degree $d$ univariate polynomials over $\mathbb{F}_q$
  - $|RS| = q^{d+1}$
  - $\Delta(RS) = q - d$
  - $q^{d+1}$ codewords in $[q]^q$ with distance $q - d$
Threshold Graph Construction

\[ A_t = \{0, 1\}^r \]

\[ U_i^0 = C \]

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Threshold Graph Construction

\[ U_0 = \{ 0, 1 \}^r \]

\[ (u, (q_1, \ldots, q_r)) \in U_i^0 \times A_t \text{ is an edge } \iff u_t = q_i \]
Completeness

For every $(u^1, \ldots, u^r) \in U_1^0 \times \cdots \times U_r^0$ and every $A_t$
there exists a unique common neighbor of $(u^1, \ldots, u^r)$ in $A_t$
Threshold Graph Properties

Completeness

For every \((u^1, \ldots, u^r) \in U_1^0 \times \cdots \times U_r^0\) and every \(A_t\) there exists a unique common neighbor of \((u^1, \ldots, u^r)\) in \(A_t\)

Soundness

For every \(u, u' \in U_i^0\), there are at most \(L - \Delta(C)\) many supernodes in \(A\) which have a common neighbor of \(u\) and \(u'\)
\[ A_t = \{0, 1\}^r \]

\[ U_i^0 = C \]
Threshold Graph Composition

\[ A_t = \{0, 1\}^r \]

\( (w, (q_1, \ldots, q_r)) \in W_j \times A_t \) is an edge if and only if there exists \( (u^1, \ldots, u^r) \in U_1^0 \times \cdots \times U_r^0 \) such that for all \( i \in [k] \), \( (w, u^i) \) and \( (u^i, (q_1, \ldots, q_r)) \) are both edges.
Completeness of Reduction

- Let \((w_1, \ldots, w_k) \in W_1 \times \cdots \times W_k\) be \textit{optimal} labeling of \(\Gamma_0\).

- Let \((u^1, \ldots, u^r) \in U^0_1 \times \cdots \times U^0_r\) be \textit{common neighbors} of \((w_1, \ldots, w_k)\) in \(\Gamma_0\).
Completeness of Reduction

- Let \((w_1, \ldots, w_k) \in W_1 \times \cdots \times W_k\) be optimal labeling of \(\Gamma_0\)
- Let \((u^1, \ldots, u^r) \in U^0_1 \times \cdots \times U^0_r\) be common neighbors of \((w_1, \ldots, w_k)\) in \(\Gamma_0\)

Completeness of Threshold Graph

For every \((u^1, \ldots, u^r) \in U^0_1 \times \cdots \times U^0_r\) and every \(A_t\)
there exists a unique common neighbor of \((u^1, \ldots, u^r)\) in \(A_t\)
Soundness of Reduction

- Fix \((w_1, \ldots, w_k) \in W_1 \times \cdots \times W_k\)
- There exists \(U_i^0\) not covered by \((w_1, \ldots, w_k)\)
Soundness of Reduction

- Fix \((w_1, \ldots, w_k) \in W_1 \times \cdots \times W_k\)
- There exists \(U_i^0\) not covered by \((w_1, \ldots, w_k)\)
- There exists \(w_j\) and \(w_{j'}\) with neighbors \(u\) and \(u'\) resp. in \(U_i^0\) \((u \neq u')\)
Soundness of Reduction

- Fix \((w_1, \ldots, w_k) \in W_1 \times \cdots \times W_k\)
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- There exists \(w_j\) and \(w_j'\) with neighbors \(u\) and \(u'\) resp. in \(U^0_i\) \((u \neq u')\)
- If \(a \in A\) is common neighbor of \(w_j\) and \(w_j'\) in \(\Gamma\) then \(u\) and \(u'\) are common neighbors of \(a\) in Threshold graph
Soundness of Reduction

- Fix \((w_1, \ldots, w_k) \in W_1 \times \cdots \times W_k\)
- There exists \(U_0^i\) not covered by \((w_1, \ldots, w_k)\)
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Soundness of Threshold Graph

For every \(u, u' \in U_0^i\), there are at most \(L - \Delta(C)\) many supernodes in \(A\) which have a common neighbor of \(u\) and \(u'\)
Inapproximability of MaxCover using Random Binary Codes

There is a FPT reduction from MaxCover instance \( \Gamma_0 = \left( U_0 = \bigcup_{j=1}^{r} U_j^0, W = \bigcup_{j=1}^{k} W_i, E_0 \right) \) with projection property to a MaxCover instance \( \Gamma = \left( U = \bigcup_{j=1}^{O(\log |U_0|)} U_j, W = \bigcup_{j=1}^{k} W_i, E \right) \) such that

- If \( \text{MaxCover}(\Gamma_0) = 1 \) then \( \text{MaxCover}(\Gamma) = 1 \)
- If \( \text{MaxCover}(\Gamma_0) < 1 \) then \( \text{MaxCover}(\Gamma) \leq 0.75 \)
- \( |\Gamma| = \tilde{O}(2^r \cdot |W| \cdot \log |U_0|) \)
- The reduction runs in time \( 2^{O(r)} \cdot \text{poly}(|\Gamma_0|) \).
Threshold Graph Composition with Reed Solomon Codes

\[ A_t = [q]^r \]

\[ U_i^0 = C \]

\[(w, (q_1, \ldots, q_r)) \in W_j \times A_t \text{ is an edge } \iff \exists (u^1, \ldots, u^r) \in U_1^0 \times \cdots U_r^0 \text{ such that} \]

\[ \forall i \in [k], (w, u^i) \text{ and } (u^i, (q_1, \ldots, q_r)) \text{ are both edges} \]
Threshold Graph Properties

Completeness

For every $(u_1, \ldots, u_r) \in U_1^0 \times \cdots \times U_r^0$ and every $A_t$ there exists a unique common neighbor of $(u_1, \ldots, u_r)$ in $A_t$.

Soundness

For every $u, u' \in U_i^0$, there are at most $\log_q |U_0|$ many supernodes in $A$ which have a common neighbor of $u$ and $u'$. 
MaxCover: Gap Creation

Inapproximability of MaxCover using Reed Solomon Codes

There is a FPT reduction from MaxCover instance $\Gamma_0 = \left( U_0 = \bigcup_{j=1}^{r} U_j^0, W = \bigcup_{j=1}^{k} W_i, E_0 \right)$ with projection property to a MaxCover instance $\Gamma = \left( U = \bigcup_{j=1}^{q} U_j, W = \bigcup_{j=1}^{k} W_i, E \right)$ such that

- If $\text{MaxCover}(\Gamma_0) = 1$ then $\text{MaxCover}(\Gamma) = 1$
- If $\text{MaxCover}(\Gamma_0) < 1$ then $\text{MaxCover}(\Gamma) \leq \frac{\log_q |U_0|}{q}$
- $|\Gamma| = \tilde{O}(q^r \cdot |W| \cdot \log |U_0|)$
- The reduction runs in time $q^r \cdot \text{poly}(|\Gamma_0|)$. 
Part 2

Gap Creation in One-Sided Biclique
One-Sided Biclique: Recap

\[ \Gamma(U, W, E) \]
One-Sided Biclique: Recap

\[ \Gamma(U, W, E) \]

Find \( k \) vertices in \( W \) with most common neighbors

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One-Sided Biclique: Recap

Find \( k \) vertices in \( W \) with most common neighbors

\[ \Gamma(U, W, E) \]
Inapproximability of One-Sided Biclique (Lin’18)

There is a FPT reduction from $k$-Clique instance $G([n], E_0)$ to a One-Sided Biclique instance $\Gamma = (U, W, E)$ such that

- If $G$ has a $k$-clique then there are $\binom{k}{2}$ vertices in $W$ which have $n^{1/k}$ common neighbors in $U$.
- If $G$ has no $k$-clique then for every $\binom{k}{2}$ vertices in $W$ they have at most $(k + 1)!$ common neighbors in $U$.
- $|\Gamma| = n^3$.
- The reduction runs in time $\text{poly}(n)$. 

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Starting from $k$-Clique

Input: $G([n], E_0)$

$V = [n]$  $W = E_0$

If $G$ has a $k$-clique then there are $(k^2)$ vertices in $W$ which in total have $k$ neighbors.

If $G$ has no $k$-clique then any $(k^2)$ vertices in $W$ has totally at least $k+1$ neighbors.
Starting from $k$-Clique

Input: $G([n], E_0)$

If $G$ has a $k$-clique then there are $\binom{k}{2}$ vertices in $W$ which in total have $k$ neighbors.
Starting from \( k \)-Clique

\[
V = [n] \\
W = E_0
\]

Input: \( G([n], E_0) \)

If \( G \) has a \( k \)-clique then there are \( \binom{k}{2} \) vertices in \( W \) which in total have \( k \) neighbors

If \( G \) has no \( k \)-clique then any \( \binom{k}{2} \) vertices in \( W \) has totally at least \( k+1 \) neighbors
Threshold Graph

\[ A = [n] \]

\[ V = [n] \]

Every \( k \) vertices in \( V \) has at least \( n - 1/k \) common neighbors in \( A \).

Every \( k + 1 \) vertices in \( V \) has at most \((k + 1)!\) common neighbors in \( A \).
Threshold Graph

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\[ V = [n] \]

Every \( k \) vertices in \( V \) has at least \( n^{1/k} \) common neighbors in \( A \)

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Threshold Graph Composition

\[ W = E_0 \]

\[ A = [n] \]

\[ V = [n] \]

\[ (w, a) \in W \times A \text{ is an edge} \iff \exists u, u' \in U \text{ such that } a \text{ and } w \text{ are common neighbors of } u \text{ and } u' \]
$W = E_0$

$A = [n]$

$V = [n]$

$(w, a) \in W \times A$ is an edge $\iff \exists u, u' \in U$ such that $a$ and $w$ are common neighbors of $u$ and $u'$.
Completeness of Reduction

- Let $v_1, \ldots, v_k \in V$ be vertices of $k$-clique in $G$
- Let $A' \subseteq A$ be common neighbors of $v_1, \ldots, v_k$ in Threshold graph
Completeness of Reduction

- Let $v_1, \ldots, v_k \in V$ be vertices of $k$-clique in $G$
- Let $A' \subseteq A$ be common neighbors of $v_1, \ldots, v_k$ in Threshold graph
- Every $a \in A'$ is also a common neighbor of $e_{v_i,v_j} \in W$ in $\Gamma$
Completeness of Reduction

- Let $v_1, \ldots, v_k \in V$ be vertices of $k$-clique in $G$
- Let $A' \subseteq A$ be common neighbors of $v_1, \ldots, v_k$ in Threshold graph
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Completeness of Threshold Graph

Every $k$ vertices in $V$ has at least $n^{1/k}$ common neighbors in $A$
Soundness of Reduction

- Fix \((w_1, \ldots, w_{k_2}) \in W\) and let \(A' \subseteq A\) be its set of common neighbors in \(\Gamma\).
Soundness of Reduction

- Fix \((w_1, \ldots, w_{\binom{k}{2}}) \in W\) and let \(A' \subseteq A\) be its set of common neighbors in \(\Gamma\).
- Let \(V' \subseteq V\) be set of total neighbors of \((w_1, \ldots, w_{\binom{k}{2}})\) in \(V\).
- \(|V'| \geq k + 1\)
Soundness of Reduction

1. Fix \((w_1, \ldots, w_{\binom{k}{2}}) \in W\) and let \(A' \subseteq A\) be its set of common neighbors in \(\Gamma\).
2. Let \(V' \subseteq V\) be the set of total neighbors of \((w_1, \ldots, w_{\binom{k}{2}})\) in \(V\).
3. \(|V'| \geq k + 1\).
4. \(A'\) is a subset of the common neighbors of \(V'\) in the Threshold graph.
Soundness of Reduction

- Fix \((w_1, \ldots, w_{k+1}) \in W\) and let \(A' \subseteq A\) be its set of common neighbors in \(\Gamma\).
- Let \(V' \subseteq V\) be set of total neighbors of \((w_1, \ldots, w_{k+1})\) in \(V\).
- \(|V'| \geq k + 1\).
- \(A'\) is a subset of the common neighbors of \(V'\) in Threshold graph.

Soundness of Threshold Graph

Every \(k+1\) vertices in \(V\) has at most \((k + 1)!\) common neighbors in \(A\).
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- If $G$ has no $k$-clique then for every $\binom{k}{2}$ vertices in $W$ they have at most $(k+1)!$ common neighbors in $U$.
- $|\Gamma| = n^3$
- The reduction runs in time $\text{poly}(n)$
Threshold Graph

$A = [n]$ \hspace{5cm} V = [n]

Every $k$ vertices in $V$ has at least $n^{1/k}$ common neighbors in $A$.

Every $k+1$ vertices in $V$ has at most $(k + 1)!$ common neighbors in $A$. 
Threshold Graph

\[ A = [n] \]

\[ V = [n] \]

Every \( k \) vertices in \( V \) has at least \( n \) common neighbors in \( A \).

Every \( k + 1 \) vertices in \( V \) has at most \((k + 1)\)! common neighbors in \( A \).

IT DOES EXIST
Random Algebraic Constructions

- Erdős-Renyi model Random graphs fail: long smooth-decaying tail
Random Algebraic Constructions

- Erdős-Renyi model Random graphs fail: long smooth-decaying tail
- Random graphs defined over some specific ‘algebraic distribution’ suffice
Random Algebraic Constructions

- Erdös-Renyi model Random graphs fail: long smooth-decaying tail
- Random graphs defined over some specific ‘algebraic distribution’ suffice
- Normed graphs provide semi-explicit construction
Take-away Intuition and Remarks

- Threshold Graph Composition Technique Ingredients:
  - Threshold Graph
  - Composition of Input Graph with Threshold Graph
Take-away Intuition and Remarks

- Threshold Graph Composition Technique Ingredients:
  - Threshold Graph
  - Composition of Input Graph with Threshold Graph
- Threshold Graph
  - What are the required threshold properties?
  - Does the graph with above properties exist?
Take-away Intuition and Remarks

- Threshold Graph Composition Technique Ingredients:
  - Threshold Graph
  - Composition of Input Graph with Threshold Graph
- Threshold Graph
  - What are the required threshold properties?
  - Does the graph with above properties exist?
- Tweak ‘Composition of Input Graph with Threshold Graph’ in order to require weaker/more realistic threshold properties
Take-away Intuition and Remarks

- Threshold Graph Composition Technique Ingredients:
  - Threshold Graph
  - Composition of Input Graph with Threshold Graph

- Threshold Graph
  - What are the required threshold properties?
  - Does the graph with above properties exist?

- Tweak ‘Composition of Input Graph with Threshold Graph’ in order to require weaker/more realistic threshold properties

- Start from more structured Input problem
Tomorrow’s plan

- Set Cover
- Biclique
- Clique